Journal of Engineering Physics and Thermophysics, Vol. 64, No. 5, 1993

AN INTEGRAL METHOD OF CALCULATING DYNAMIC CHARACTERISTICS OF A TURBULENT JET OUTFLOWING FROM A CURVILINEAR SLOTTED CHANNEL INTO A SUBMERGED SPACE

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UDC 532.517.408

Calculation results and comparison with the experimental data on a free submerged jet flowing out of a curvilinear slotted channel are presented. A unique equation for Monin-Obukhov's coefficient valid for wall flows and free turbulence problems is obtained.

It is known [1] that the initial nonuniformity of a velocity profile is accompanied by elevated turbulence level and turbulent viscosity as against the initial uniformity of the profile. Moreover, the curvilinear surface effect causes a change in the turbulence behavior due to the violation of the balance between centrifugal and pressure forces in jets and wakes [2]. The longitudinally oriented Taylor-Hertler vortices formed near the concave wall of a slotted channel owing to the available centrifugal forces [3] is another specific feature. In particular, this results in the fact that the averaged velocity distribution is characterized by nonsymmetry relative to the central jet plane, and the velocity maximum is shifted to the side of the concave channel wall [4] outward from the initial cross section. The distribution of rms values of longitudinal velocity component pulsations is clearly nonsymmetric. This is attributed to different conditions of turbulence generation in a given jet cross section [5].

The above-mentioned specific features presuppose use of turbulent boundary layer equations to a second approximation which take account of the variable streamline curvature in an explicit form and modification of the turbulent viscosity model supplemented with Monin-Obukhov's coefficient and the Richardson criterion [6]. The present article is aimed at developing an integral method of calculating nonsymmetric curvilinear jet flows based both on the integral relations of boundary layer theory to a second approximation and on the method of polynomial approximation of an expression for turbulent friction stress [7].

The integral relations of turbulent jet theory for a jet outflowing from a curvilinear slotted channel into a submerged space are written as [2]

$$\frac{d}{ds}\int_0^{b_1} u^2 dn = 0, \quad \frac{d}{ds}\int_0^{-b_2} u^2 dn = 0, \tag{1}$$

$$\frac{d}{ds}\int_0^{b_1} (uh)^2 u dn = -\frac{2b_1}{R}\varkappa u_m^3 - \frac{2}{\rho}\int_0^{b_1} \tau h^2 \frac{\partial}{\partial n}(uh)\, dn. \tag{2}$$

Here, integration is made from the line of the maximum values of velocity ($n = 0$) to the nonsymmetric boundaries of the jets b_1 and b_2, respectively (Fig. 1). Equations (1) show that as in the case of a rectilinear submerged jet the equal-momentum condition remains invariable if this condition is considered along the curved line of the maximum velocity values. System of equations (1), (2) in the limiting case of a rectilinear symmetric jet ($R \to \infty$, $b_1 = b_2$) reduces to known relations [1].

Solving the problem by the integral method relies on assigning a velocity profile in the following form:

$$u/u_m = (1-\eta)^3(1+3\eta) - \frac{b_1}{R}\eta(1-\eta)^3, \tag{3}$$

here $\eta = (n - n_{01})/b_{01}$ for the entrance section and $\eta = n/b_1$ for the main section. Turbulent friction stress is modeled

St. Petersburg State Technical University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 64, No. 5, pp. 537-542, May, 1993. Original article submitted May 13, 1992.

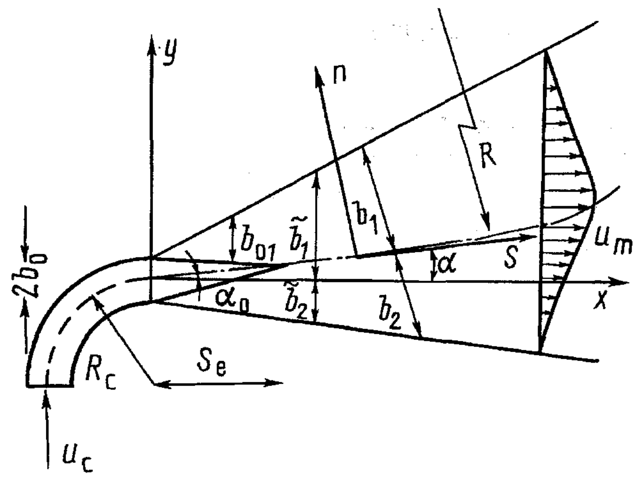


Fig. 1. Flow pattern of a turbulent jet outflowing from a curvilinear slotted channel into a submerged space.

by using the Prandtl formula and the Bradshow correction function γ_r [6] for the longitudinal streamline curvature

$$\tau = \varkappa \rho u_m b_1 \frac{\partial u}{\partial n} \gamma_r, \quad \gamma_r = 1 - \beta \overline{\mathrm{Ri}}, \tag{4}$$

where Monin-Obukhov's constant β and turbulence structures κ are determined from the best agreement between the calculated and experimental data; $\overline{\mathrm{Ri}} = \int_{-b_2}^{b_1} \mathrm{Ri}\, dn/(b_1+b_2)$ is the jet cross-section-mean Richardson number; $\mathrm{Ri} = 2(u/R)/(\partial u/\partial n)$ is the local Richardson number. When the velocity profile is predetermined by expression (3) $\overline{\mathrm{Ri}} = -4/5(b_1+b_2)/R$.

To determine jet flow characteristics seven parameters must be calculated; therefore, to close the system of equations (1), (2) the latter are supplemented with four equations, three of which are found by the differential geometry formulas

$$R = ds/d\alpha, \quad \cos\alpha = dx/ds, \quad \sin\alpha = dy/ds \tag{5}$$

and by the velocity conjugation condition on the jet axis [2]

$$b_1 = b_2\left(2 - \frac{b_1}{R}\right) \Big/ \left(2 + \frac{b_2}{R}\right). \tag{6}$$

Based on expressions (3) and (4) for the velocity profile and turbulent friction, respectively, we calculate the values of the integrals entering into (1), (2), which reduce to a system of ordinary differential equations for the entrance section

$$\frac{dn_{01}}{ds} = -\frac{2a_2a_4}{(a_2 - a_3)} \varkappa \left[1 - \frac{2}{5}\beta \frac{(b_{01} + b_{02})}{R}\right], \tag{7}$$

$$\frac{db_{01}}{ds} = \frac{2a_4}{(a_2 - a_3)} \varkappa \left[1 - \frac{2}{5}\beta \frac{b_{01} + b_{02}}{R}\right], \tag{8}$$

$$\frac{db_{02}}{ds} = \frac{2a_4}{(a_2 - a_3)} \varkappa \left[1 + \frac{2}{5}\beta \frac{b_{01} + b_{02}}{R}\right] \tag{9}$$

under the initial conditions

$$s = 0, \ n_{01} = b_0, \ b_{01} = b_{02} = 0. \tag{10}$$

For the main section the system of equations is written as

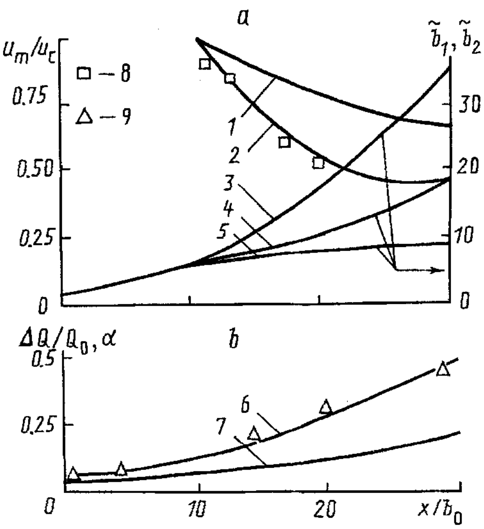


Fig. 2. Characteristic thickness and maximum velocity variations in a jet; 8) data from [5] (a) and calculations of velocity maximum vs deviation angle (experiment 9, data from [2]) and relative flowrate variations (b). α, deg.

$$\frac{du_m}{ds} = -2\frac{a_4}{a_3}\varkappa\frac{u_m}{b_1}\left[1 - \frac{2}{5}\beta\frac{(b_1 + b_2)}{R}\right], \tag{11}$$

$$\frac{db_1}{ds} = 4\frac{a_4}{a_3}\varkappa\left[1 - \frac{2}{5}\beta\frac{(b_1 + b_2)}{R}\right], \tag{12}$$

$$\frac{db_2}{ds} = 4\frac{a_4}{a_3}\varkappa\frac{b_2}{b_1}\left[1 + \frac{2}{5}\beta\frac{(b_1 + b_2)}{R}\right] \tag{13}$$

subject to the corresponding initial conditions

$$s = s_e,\ u_m = u_c,\ b_1 = b_{1e},\ b_2 = b_{2e}. \tag{14}$$

Values of the constants a_j ($j = 1, 2, 3, 4$) entering into the systems of equations (7)-(9) and (11)-(13) and representing the values of the integrals of the velocity profiles are taken from [8]. The system of equations (7)-(9) permits an analytical solution for the characteristics of a nonsymmetric jet over the entrance section

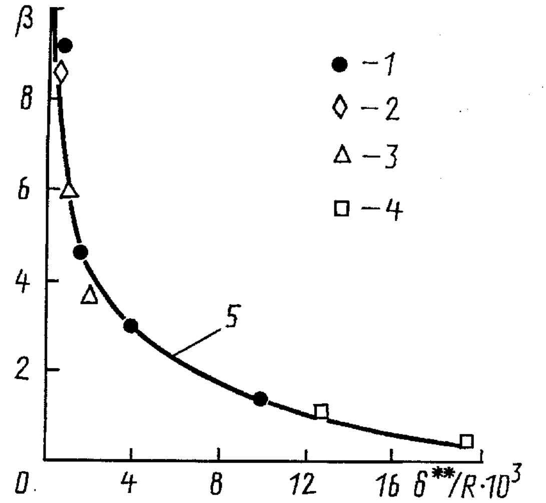
$$b_{01,2} = \frac{2a_4\varkappa}{(a_2 - a_3)}\left[1 \mp \sqrt{1 - \frac{4}{5}\beta}\right]s, \tag{15}$$

$$n_{01} = b_0 - \frac{2a_4a_2}{(a_2 - a_3)}\left[1 - 2\sqrt{1 - \frac{4}{5}\beta}\right]\varkappa s, \tag{16}$$

$$R = \frac{4}{5}\frac{\varkappa a_4\beta}{(a_2 - a_3)}s \Bigg/ \left[1 - \frac{4}{5}\beta\right]. \tag{17}$$

The system of equations (11)-(13) for the main section was solved numerically by the Runge-Kutta method. In this case, the boundaries $\tilde{b}_1$ and $\tilde{b}_2$ of the turbulent mixing region were reckoned from the straight line normal to the nozzle exit section (Fig. 1).

Analysis of expression (3) shows that when affected by the curvilinear surfaces the velocity profile becomes less full, especially over the range $0 < \eta \leq 0.5$. Damping of the dimensionless maximum velocity in the nonsymmetric jet occurs more rapidly (curve 2, Fig. 2a) as against the jet outflowing from a flat nozzle with a rectilinear axis (curve 1). This difference is 32%. Here, the mixing region thicknesses $\tilde{b}_1$ and $\tilde{b}_2$ (curves 3, 4, respectively) are presented and compared with the opening of a flat submerged jet (curve 5). Figure 2b plots the results for the velocity maximum calculated in terms of the deviation angle from the central plane α and for the relative flowrate $\Delta\dot{Q}/\dot{Q}_0$ variation ($\dot{Q}_0$ is the flowrate in a symmetric submerged jet). The best coincidence between the calculated and experimental data on maximum velocity damping [5], angle α, and entrance section length s_e [2] was



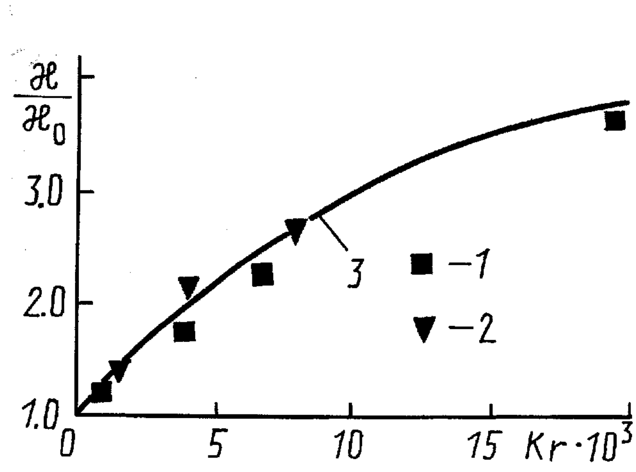


Fig. 3. Plot of Monin-Obukhov's coefficient: 1) data of [9]; 2) [11]; 3) [2]; 4) data of the present authors; 5) relation (18).

Fig. 4. Turbulent structure coefficient vs curvature parameter: 1) data [2]; 2) data of the present authors; 3) relation (18).

attained at the following values of the constants β and κ for the entrance and main sections, respectively: $\beta = 1.245$; 0.30; $\kappa = 0.0083$; 0.04.

From Fig. 2b it follows that with increasing distance from the nozzle exit section the velocity profile deforms, resulting in the fact that the velocity maximum is shifted to the side of the concave nozzle surface (curve 6). The relative variation of the flowrate $\Delta\dot{Q}/\dot{Q}_0$ grows with increasing distance from the nozzle exit section (curve 7, Fig. 2b), thereby pointing to the growth of the ejection abilities of such jets.

Comparison of the experimental and calculated results allows a generalization of the available values of Monin-Obukhov's coefficient β for jets and mixing layers where the mixing layer momentum loss thickness-to-local curvature radius ratio δ^{**}/R (surfaces for a wall jet, external flow streamlines for a free concurrent jet, and maximum velocity lines for a submerged jet) is chosen as the curvature parameter. For boundary layer-type flows δ^{**}/R uniquely characterizes coefficient β variation over the range $0 < \delta^{**}/R < 0.01$ [9]. A similar approach is proposed to write the coefficient κ, where the quantity $K_r = (1-u_l/u_m)(\delta^{**}/R)$ is used as a generalizing complex, which has been obtained for the first time from the asymptotic analysis of the integral relations and points to different action of centrifugal forces in jets and wakes [10].

Generalization of the calculated and experimental data has enabled one to plot a unique relation (curve 5, Fig. 3) for Monin-Obukhov's coefficient β for jet flows over a wider range of the curvature parameter $0 < \delta^{**}/R < 0.02$, which also includes the recommendations used for boundary layer calculations [11]. Figure 4 plots the turbulent structure coefficient κ based on the corresponding value of κ_0 for a symmetric jet [1]. Criterial equations that approximate the experimental data for the constants β and κ can be written within ±7% in the following form:

$$\beta = 9\left[1 + 8000\left(\frac{\delta^{**}}{R} - 0{,}001\right)\right]^{-0{,}6}, \quad \varkappa/\varkappa_0 = 1 + 73{,}4K_r^{0{,}8}. \qquad (18)$$

The semiempirical calculation method presented in this article and expressions (18) make it possible to allow for the action of curvilinear surfaces upon the mixing processes in gas jets. This is necessary in forming a lower blast jet which is used for arranging multiple circulation of solid fuel particles in a low-temperature vortex furnace [12].

NOTATION

s, n, coordinates of a curvilinear coordinate system associated with the maximum velocity line (s-axis, n, normal to it); x, y, Cartesian system coordinates; b_1 and b_2, thicknesses of a nonsymmetric submerged jet reckoned from the maximum velocity line; $\bar{b}_1 = b_1\cos\alpha + S\sin\alpha$, $\bar{b}_2 = \cos\alpha - S\sin\alpha$, mixing boundaries determined from the straight line perpendicular to the nozzle exit section; u, longitudinal averaged velocity component; u_m, u_l, maximum velocity in a given cross section and concurrent flow velocity; $\delta^{**} = \int_0^{b_1} u/u_m(1-u/u_m)dh$ momentum loss thickness of the mixing region; $R = R(s)$, local curvature; $h = 1+n/R$, Lamé coefficient; n_{01}, jet "core" halfwidth on the entrance section; R_c, curvature radius of the central line of the slotted nozzle; $\dot{Q} = \int_{-b_2}^{b_1} u dn$, volume flowrate per second; $\Delta\dot{Q} = \dot{Q} - \dot{Q}_0$, second flowrate variation.

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